A Brief Tutorial and Survey on Markovian Jump Systems

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Markovian jump systems (MJSs) can be regarded as a special type of jump system, whose jumping law governing the switches among the subsystems are a Markovian chain or process [1]. Similar to other control systems, the subsystems in MJSs are usually described by some type of dynamic equations, while a Markov process that can be either continuous time or discrete time describes the jumping law. On the other hand, MJSs also are hybrid dynamic systems typically consisting of both the dynamic state space and the set of discrete

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events, where a Markov process describes the discrete events for MJSs [2], [3].

Other than their own theoretical and practical importance, by their nature, MJSs provide a powerful tool for modeling and controlling various practical systems such as, networked control systems [4], [5], manufacturing applications [6]–[8], economics systems [9]–[11], power engineering [13], [14], aerospace engineering [12], and communication systems [15], [16]. Many engineering systems may experience sudden switches of their working points, due to sudden failure of system components or interconnection parts, abrupt environmental disturbances that may drive the working point away, or nonlinearity of the plant that may lead to a leap of the working point. We see that these switches of the working points are either practically memoryless, i.e., the current switch does not depend on the switches from a long time ago, or historically dependent, but it is simply too difficult or unnecessary to include the historical dependence in the model. Therefore, the switches are often assumed to be Markovian and hence result in an MJS.

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Based on its theoretical and practical importance, the study of MJSs has attracted a lot of attention from the control community since its first introduction in 1961 [1], and are still prominent today. Researchers have borrowed many concepts, tools, and methods from other control domains to study the tracking, stability, optimization, and fault tolerance of MJSs and have yielded fruitful results [17], [20]–[23]. Undoubtedly, however, the theoretical development of MJSs has its own unique challenges, thanks to the existence of the exclusive Markovian jumping law. Such a law has produced several seemingly impossible system behaviors, such as when the stability of all the subsystems does not guarantee the stability of the whole system, and when the instability of all the subsystems also may not prevent the stability of the whole system. In addition, dealing with delays, nonlinearity, noises, disturbances, modeling errors, filtering, robustness, optimal control, adaptive control, and many other control problems are also core in developing MJSs theory. Many interesting results have been obtained in the past several decades [18], [19].

We provide a brief tutorial and survey of the stability analysis and control approaches for MJSs. The scope is not comprehensive; it focuses only the stability and control aspects, of all the possible discussion points of MJSs. We will first explain the concepts and definitions, for the benefit of the newcomers to the field, and then introduce state-of-the-art recent developments. We hope readers find this tutorial and survey useful.

Notations and Definitions

Throughout this article, the vectors are in their column form unless otherwise explicitly specified and a superscript T is placed for the transpose of vectors and matrices. $\mathbb{R}_+, \mathbb{R}^n$, and $\mathbb{R}^{n\times m}$ are for the set of nonnegative real numbers, n-dimensional real space, and $n\times m$ dimensional real matrix space, respectively. $|a|$ is the Euclidean norm of $a\in\mathbb{R}^n$, i.e., $|a|=(\Sigma_{i=1}^n a_i^2)^{1/2}$. $C([-\mu,0];\mathbb{R}^n), C^i$, and $C^{i,k}$, respectively, denote the continuous $\mathbb{R}^n$-valued function space defined on $[-\mu,0]$, the set of the ith continuous differential functions, and the set of functions with the ith first component and the kth second component being continuously differentiable. For stochastic variable x, $E\{x\}$ is its expectation. $\psi\circ\varphi: A\to C$ is the composition of $\varphi: A\to B$ and $\psi: B\to C$.

We define several function classes as follows.

1) Class $\mathcal{K}$ function $\varphi(u)$ is strictly increasing in u and $\varphi\in C(\mathbb{R}_+,\mathbb{R}_+), \varphi(0)=0$.
2) Class $\mathcal{K}_\infty$ functions contain only those that are unbounded.
3) Class $\mathcal{KL}$ function $\beta:\mathbb{R}_+\times\mathbb{R}_+\to\mathbb{R}_+:\beta(s,t)$ decreases to 0 as $t\to+\infty$ for each fixed $s\geq 0$ and $\beta(\cdot,t)$ is of class $\mathcal{K}$ in the first argument for each fixed $t\geq 0$.
4) Class generalized $\mathcal{K}$ ($\mathcal{GK}$) function $h:\mathbb{R}_+\to\mathbb{R}_+$ continuous with $h(0)=0$ and satisfies

$$\begin{cases} h(r_1)>h(r_2), & \text{if } h(r_1)\neq 0; \\ h(r_1)=h(r_2)=0, & \text{if } h(r_1)=0, \end{cases} \quad \forall r_1>r_2. \tag{1}$$

The following facts hold: 1) A class $\mathcal{GK}$ function is a (conventional) class $\mathcal{K}$ function and 2) a function $\beta:\mathbb{R}_+\times\mathbb{R}_+\to\mathbb{R}_+$ is a $\mathcal{GKL}$ function if for each fixed $t\geq 0, \beta(s,t)$ is a generalized $\mathcal{K}$-function and for each fixed $s\geq 0$ it decreases to zero as $t\to T$ for some $T\leq\infty$.

Stability of MJSs

This section reviews multiple stability notions for MJSs, each of which has its own values. We start from Lyapunov stability and then review in sequential input-to-state stability, practical stability, and finite-time stability. For each stability, we discuss the definition and the criteria for ensuring the stability, as well as the recent development of the corresponding stability analysis in the literature.

A Lyapunov Stability

Lyapunov stability is perhaps the most commonly used stability notion, which is determined by whether the system equilibrium point can be kept under small perturbations. We discuss Lyapunov stability for linear and nonlinear MJSs in what follows.

Lyapunov Stability of Linear MJSs

Consider the linear MJS

$$\begin{cases} \dot{x}(t) = A(r(t))x(t), & t \geq 0 \\ x(0) = x_0 \in \mathbb{R}^n, \end{cases} \tag{2}$$

where $r(t)$ is a continuous-time discrete-state Markov process. The state space of $r(t)$ is $\mathcal{S} = \{1, 2, \ldots, N\}$ and the transition probability from state i to j, i.e., p_{ij}, is given as

$$\begin{aligned} p_{ij} &= Pr(r(t+\Delta) = j \mid r(t) = i) \\ &= \begin{cases} \pi_{ij}\Delta + o(\Delta), & i \neq j; \\ 1 + \pi_{ii}\Delta + o(\Delta), & i = j, \end{cases} \quad \Delta > 0 \end{aligned}$$

with $\pi_{ij} \geq 0$ being the transition rate from state i to $j (i \neq j)$ and $\pi_{ii} = -\Sigma_{j=1, j\neq i}^{N} \pi_{ij}$.

Definition 1: Lyapunov Stability of Linear MJSs

The equilibrium point 0 for the system in (2) is as follows:

1) asymptotically mean square stable, if $\forall x_0 \in \mathbb{R}^n$ and for any initial distribution $(p_1, \ldots, p_N)$ of $r(t)$ such that

$$\lim_{t\to+\infty} E\{\|x(t, x_0, \omega)\|^2\} = 0 \tag{3a}$$

2) exponentially mean square stable, if $\forall x_0 \in \mathbb{R}^n$ and for any $(p_1, \ldots, p_N)$ of $r(t)$, there exists constants $\alpha, \beta > 0$ such that

$$E\{\|x(t, x_0, \omega)\|^2\} \leq \alpha \|x_0\|^2 e^{-\beta t}, \quad \forall t \geq 0 \tag{3b}$$

3) stochastically stable, if $\forall x_0 \in \mathbb{R}^n$ and for any $(p_1, \ldots, p_N)$ of $r(t)$ such that

$$\int_0^{+\infty} E\{\|x(t, x_0, \omega)\|^2\} dt < +\infty \tag{3c}$$

4) almost surely (asymptotically) stable, if $\forall x_0 \in \mathbb{R}^n$ and for any $(p_1, \ldots, p_N)$ of $r(t)$ such that

$$P\Big\{\lim_{t\to+\infty} \|x(t, x_0, \omega)\| = 0\Big\} = 1. \tag{3d}$$

In Definition 1, the stability definitions in 1)–3) are equivalent, and they all imply 4) [24].

The stochastic stability conditions for the system in (2) proposed in Theorem 1 are sufficient and necessary. Therefore, these conditions are also adequate and necessary for asymptotically mean-square stability and exponentially mean-square stability, and are almost surely sufficient for (asymptotically) stability.

Theorem 1: Lyapunov Stability Criteria for Linear MJSs

The system in (2) is stochastic stable if and only if there exists matrices $P_i, i \in \mathcal{S}$ such that

$$A_i^T P_i + P_i A_i + \mathcal{P}_i < 0, \tag{4}$$

where $\mathcal{P}_i = \Sigma_{j\in\mathcal{S}} \pi_{ij} P_j$ and $A_i = A(r_t), r_t \in \mathcal{S}$. The stability definitions and criteria for the linear discrete-time MJS

$$x(k+1) = A(r_k)x(k) \tag{5}$$

can be obtained similarly. Due to the space restrictions of this article, we will not provide details here; however, interested readers may refer to [25] and [26] for further information.

Lyapunov Stability of Nonlinear MJSs

Consider the following stochastic differential equation with Markovian switching [26]:

$$\begin{aligned} &dx(t) = f(x(t), r(t))dt + g(x(t), r(t))dB(t), \quad t \geq t_0 \\ &x(t_0) = x_0 \end{aligned} \tag{6}$$

with solutions defined on $t \geq t_0$, initial values $x_0 \in \mathbb{R}^n$ and $r_0 \in \mathcal{S}$. Here $f(\cdot): \mathbb{R}^n \times \mathcal{S} \to \mathbb{R}^n$, $g(\cdot): \mathbb{R}^n \times \mathcal{S} \to \mathbb{R}^n$ and the m-dimensional Brownian motion $B(\cdot)$ is defined on $(\Omega, \mathcal{F}, P)$ and is independent of $r(t)$. Both functions $f(\cdot)$ and $g(\cdot)$ are local Lipschitz and, consequently, the solution to (6) is unique.

Definition 2: Lyapunov Stability of Nonlinear MJSs

The equilibrium point of the system in (6) is as follows:

1) stochastically stable, if $\forall x_0$ and $t_0 \geq 0$, there exists $\rho > 0$ and $\varepsilon \in (0, 1)$ such that

$$Pr\{|x(t, t_0, x_0, r_0)| < \rho, \quad \text{for all } t \geq t_0\} \geq 1 - \varepsilon \tag{7a}$$

2) stochastically asymptotically stable in the large, if it is stochastically stable and, moreover

$$Pr\{\lim_{t\to\infty} x(t, t_0, x_0, r_0) = 0\} = 1 \tag{7b}$$

3) almost surely exponential stable, if $\forall r_0$ and $t_0 \geq 0$

$$\limsup_{t\to\infty} \frac{1}{t} \log(|x(t, t_0, x_0, r_0)|) < 0 \tag{7c}$$

4) pth moment stable, if $\forall r_0$ and $t_0 \geq 0$, there exists $\varepsilon > 0$ such that

$$E\{|x(t, t_0, x_0, r_0)|^p\} < \varepsilon \tag{7d}$$

5) exponentially stable in mean square, if constants $\epsilon_1 > 0$ and $\epsilon_2 > 0$ exist and $\forall t \geq 0$

$$E\{|x(t,t_0,x_0,r_0)|^2\} < \epsilon_1 |x_0|^2 \exp(-\epsilon_2 t) \tag{7e}$$

6) globally asymptotic stability in probability [27]; if for any given $\varepsilon > 0$, a $\mathcal{KL}$ function $\beta(\cdot,\cdot)$ exists satisfying

$$P\{|x(t,t_0,x_0,r_0)| < \beta(|x_0|,t)\} < 1-\epsilon. \tag{7f}$$

Lyapunov function and the comparison principle are often used to derive the stability criteria for nonlinear MJSs. Due to the various forms of nonlinearity, we do not discuss the detailed stability criteria but ask readers to refer to [28] and [29] and the references therein for further details.

Moreover, constraints like time delays, uncertainty, and incomplete information are often met in practice. Many efforts have been made to deal with such problems, such as stochastic differential delay equations with Markovian switching [30]–[32], MJSs with mode-dependent time-varying delays [33], linear MJSs with incomplete transition descriptions [18], [19], and linear uncertain MJSs with mode-dependent time delays [34], just to name a few.

Input-to-State Stability

With regard to nonlinear systems with external inputs, the stability notions of input-to-state stability (ISS), input-to-output stability (IOS), and integral input-to-state stability have been developed, with fruitful results obtained in recent years [35]–[44]. ISS is often used to aid the design of smooth controllers or to deal with various uncertainties that arise from applications. Many developments have been reported for various system settings [45]–[47].

In this section, we provide the general definition of stochastic ISS and the corresponding criteria. Readers may refer to [27] and the references therein for more information.

Consider the MJSs

$$dx(t) = f(x(t), t, r(t), u(t))dt + g(x(t), t, r(t), u(t))dw(t), t \geq 0, \tag{8}$$

where $x \in \mathbb{R}^n, u \in \mathbb{R}^m$ and $x_0 \in \mathbb{R}^n$ are the system state, the input, and the initial state, respectively, and the Markov process $r(t)$ is defined as in (2). A unique solution to the system, $x(\theta, x_0)$, i.e., $E(\sup_{0\leq\theta\leq t}|x(\theta,x_0)|^l) < \infty, \forall t \geq 0$, $l \geq 0$, is guaranteed by smooth enough $f: \mathbb{R}^n \times \mathbb{R}_+ \times \mathcal{S} \times \mathbb{R}^m \to \mathbb{R}^n$ and $g: \mathbb{R}^n \times \mathbb{R}_+ \times \mathcal{S} \times \mathbb{R}^m \to \mathbb{R}^{n\times r}$ [48]. On the complete probability space $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t\geq 0}, P)$, the r-dimensional Brownian motion $w(t)$ is defined, where $\Omega, \mathcal{F}$, $\{\mathcal{F}_t\}_{t\geq 0}$ and P are the sample space, the σ-field, the filtration, and the probability measure, respectively.

Definition 3: Stochastic ISS of MJSs

The system in (11) is stochastic ISS (SISS) if $\forall \varepsilon > 0$, there exists a $\mathcal{K}$ function $\gamma(\cdot)$ and a $\mathcal{KL}$ function $\beta(\cdot,\cdot)$, such that $\forall t \geq 0$ and $\forall x_0 \in \mathbb{R}^n$

$$P\{|x(t)| < \beta(|x_0|, t) + \gamma(\|u\|_{[0,t)})\} \geq 1 - \varepsilon,$$

where

$$\|u(s)\| = \inf_{\mathcal{A}\subset\Omega, P(\mathcal{A})=0} \sup\{|u(\omega,s)| : \omega \in \Omega \backslash \mathcal{A}\},$$
$$\|u\|_{[0,t)} = \sup_{s\in[0,t)} \|u(s)\|. \tag{9}$$

Theorem 2: SISS Criteria for MJSs

The system in (8) is SISS if there exists a function $V(x,t,i) \in C^{2,1}(\mathbb{R}^n \times \mathbb{R}_+ \times \mathcal{S}; \mathbb{R}_+)$ and functions $\bar{\alpha}_1, \bar{\alpha}_2, \psi \in \mathcal{K}_\infty$ such that $\forall (x,t,i) \in \mathbb{R}^n \times \mathbb{R}_+ \times \mathcal{S}$ and $u \in \mathbb{R}^m$ such that

$$\bar{\alpha}_1(|x|) \leq V(x,t,i) \leq \bar{\alpha}_2(|x|) \tag{10a}$$

$$\mathcal{L}V(x,t,i) \leq -\lambda_0 V(x,t,i) + \psi(\|u(s)\|), \tag{10b}$$

where $\mathcal{L}$ is the infinitesimal generator.

Practical Stability

Many practical applications may be asymptotically unstable in the Lyapunov sense, while the trajectory can stay within a certain desired region despite possible acceptable fluctuation, e.g., the acceptable oscillation of an aircraft or a missile. To describe such a situation, the concept of practical stability was introduced [49]. Such a stability notion was also demonstrated to be more suitable and desirable in practice under certain conditions by examples [50]. One particular advantage of practical stability is that it can describe not only the qualitative behaviors of the system but also its quantitative properties, including the transient behavior and the trajectory bounds, making such a stability notion useful in many situations [51]–[54]. Many further developments have also been realized. These include mean-square practical stability for stochastic large-scale dynamical systems [55], practical stability in probability for regime-switching diffusions [56], practical stability in the pth mean and practical stability in probability for hybrid parabolic systems with Markovian regime switching [57], and the practical controllability and optimal practical control for MJSs with time-delays [58], to name a few.

Take the following time-delayed MJS as an example [58]

$$dx(t) = f(x(t), x(t-\tau(t)), t, r(t))dt + g(x(t), x(t-\tau(t)), t, r(t))dw(t), t \geq 0, \tag{11}$$

where $\{x(\theta): -2\mu \leq \theta \leq 0\} = \xi \in C^b_{\mathcal{F}_0}([-2\mu, 0]; \mathbb{R}^n)$, $\tau(t): \mathbb{R}_+ \to [0, \mu]$ is a Borel measurable function and $r(t)$, $f(\cdot)$, $g(\cdot)$, $w(t)$ are defined in (2).

Definition 4: Practical Stability for MJSs

The system in (11) is said to be practically stable in probability (PSiP), if $\forall \delta > 0$, there exits λ with $0 < \lambda < \rho$, making that

$$P\{|x(t, t_0, \xi)| \geq \rho\} < \delta, \forall t \geq t_0 - \mu, \tag{12}$$

which holds for some $t_0 \in \mathbb{R}_+$ and $\forall \xi$ with $E\|\xi\| < \lambda$.

Moreover, the notion of uniformly PSiP can be defined similarly if the characteristic of PSiP is uniform for all $t_0 \in \mathbb{R}_+$. The proof of the relevant stability criteria usually takes advantage of the comparison principle, which is judged by the property of a deterministic system; see [58] for more details.

Finite-Time Stability

Nonsmooth control can lead to high tracking precision, fast response and disturbance rejection, and the ability to reach the target in finite time [59]–[64]. Such a notion has also been applied to MJSs [65]–[67].

The finite-time globally asymptotically stability is considered in [68]. For the system in (6), define the stochastic settling time function as $T_0(x_0, t_0, r_0, w) = \inf\{T \geq t_0 : x(t) = x(t, x_0, t_0, r_0) = 0, \forall t \geq T\}$.

Definition 5: Finite-Time Stability for MJSs

Equilibrium point 0 of the system in (6) is finite-time globally asymptotically stable in probability (FGSP). If $\forall \varepsilon > 0$ there exists a class $\mathcal{GKL}$ function $\beta(\cdot,\cdot)$ such that

$$P\{|x(t)| < \beta(|x_0|, t - t_0)\} \geq 1 - \varepsilon, \forall t \geq t_0, \forall x_0 \in \mathbb{R}^n \setminus \{0\} \tag{13}$$

and the stochastic settling time function $T_0 < +\infty, a.s.$

Theorem 3: Finite-Time Stability Criteria for MJSs

The system in (6) is FGSP if there exists a Lyapunov function $V \in C^{2,1}(\mathbb{R}^n \times [t_0, \infty) \times \mathcal{S}, \mathbb{R}_+)$, class $\mathcal{K}_\infty$ functions $\alpha_i, \bar{\alpha}_i (i = 1, \ldots, N)$ such that for some $c_i > 0, 0 < a_i < 1$, and $\forall x \in \mathbb{R}^n, t \geq t_0$ such that

$$\alpha_i(|x|) \leq V(x, t, i) \leq \bar{\alpha}_i(|x|) \tag{14a}$$

$$\mathcal{L}V(x, t, i) \leq -c_i V^{a_i}(x, t, i). \tag{14b}$$

Control of MJSs

We introduce several control approaches to MJSs as well as their recent advancements, including state feedback control, optimal control, and sampled-data control. Readers can refer to [28], which includes a review where H_2 and H_∞ performance analysis, filtering, feedback control, and sliding mode control are covered. Also consult [69]–[76] for the stabilization of MJSs, [20] for linear quadratic control theory of MJSs, [77] and [78] for the H_2 control theory of MJSs, [79]–[82] for the H_∞ control theory of MJSs, and [83]–[85] for the H_∞ filtering theory of MJSs.

A State Feedback Control

Consider the following Markovian jump linear control system where r_t is Markovian:

$$\dot{x}(t) = A(r_t)x(t) + B(r_t)u(t). \tag{15}$$

The state feedback control problem is to find a proper controller gain $K(r_t)$ to ensure the closed-loop stability where the following form of controller is implemented [20]:

$$u(t) = K(r_t)x(t). \tag{16}$$

Parameter disturbance or model uncertainties may also be considered, resulting in the robust stabilization problem. For the stabilization or robust stabilization of MJSs, the Lyapunov function method combined with linear matrix inequalities are often the effective tools; refer to [69]–[76] for more details.

Optimal Control

Optimal control has also been investigated extensively for MJSs, including, e.g., quadratic control, H_2 control, and H_∞ control.

Problem 1: Jump Linear Quadratic Optimal Control Problem [20]

The jump linear quadratic optimal control problem for the system in (15) is to minimize

$$J(t_0, x(t_0), r(t_0), T, u) = E\left\{\int_{t_0}^{T} [x^T(t)Q(r(t))x(t) + u^T(t)R(r(t))u(t)]dt \,|\, x(t_0), r(t_0)\right\}$$

over form-dependent control laws $\psi \in \Psi$

$$u(t) = \psi(t, x(t), r(t)), \psi : [t_0, T] \times \mathbb{R}^n \times \mathcal{S} \to \mathbb{R}^m, \tag{17}$$

where for some constant k (depending on ψ)

$$\|\psi(t, x, r) - \psi(t, \tilde{x}, r)\| \leq k\|x - \tilde{x}\|, \psi(t, x, r) \leq k(1 + \|x\|)$$

for all $t, x, \tilde{x}, r, R$ and Q are real valued symmetric matrices with $R(r(t)) > 0$ and $Q(r(t)) \geq 0$. T may be finite or infinite.

The discrete-time counterpart of the aforementioned problem can also be defined [86], [87]. Other problems, such as the constrained quadratic control of discrete-time linear MJSs [78], finite horizon quadratic optimal control problem, and the separation principle for linear MJSs [88] have also been considered.

To introduce the H_∞ control theory for MJSs, we first give the following Markovian jump control system with disturbance input:

$$\dot{x}(t) = A(r(t))x(t) + B(r(t))u(t) + G(r(t))w(t) \tag{18a}$$

$$z(t) = C(r(t))x(t) + D(r(t))u(t), \tag{18b}$$

where $r(t)$ is a Markov process defined as in (2); $x(t)$, $u(t)$, $w(t)$, and $z(t)$ are the system state; the control input satisfying (17); the disturbance input in $l_2[0,\infty)$; and the controlled output in $l_2[0,\infty)$.

Problem 2: H_∞ Control for MJSs [80]

The H_∞ control for the system in (18) is to design a controller as in (17), such that for all nonzero $w(t) \in l_2[0,\infty)$ it holds that

$$\|z(t)\|_{E_2} < \gamma \|w(t)\|_2, \tag{19}$$

where

$$\|z(t)\|_{E_2} = E\left\{\int_0^T z^T(t)z(t)dt\right\}^{1/2}$$

and $\gamma > 0$ is a prescribed level of disturbance attenuation. The system in (18) with the controller in (17) is said to have H_∞ performance (19) over the horizon $[0,T]$ if (19) holds.

H_∞ control theory has been intensively investigated [79]–[82]. For example, the H_∞ controller for MJSs was designed for unknown nonlinearities in [80], bounded transition probabilities in [81], and uncertainties and time delay in [79], respectively. Delay-dependent H_∞ control has also been studied for MJSs with time-varying delays [5], [82], [89]. In [90] and [91], finite-time H_∞ fuzzy control of nonlinear delayed MJSs with partly uncertain transition descriptions was discussed for discrete-time and continuous-time cases, respectively. H_∞ control for fuzzy MJSs under different conditions can be found in [92] and [93]. For 2D continuously delayed MJSs with partially unknown transition probabilities, H_∞ control [94] and the robust H_∞ filtering problem have also been considered. For example, in [69], a mode-independent filter was designed for MJSs with H_∞ performance. For more results, refer to [95]–[97].

Besides H_∞ performance, H_2 performance [98], [99] and $L_2 - L_\infty$ (energy-to-peak) performance [100], [101] are also important indices and have been investigated extensively. Due to page limitations, we do not introduce them in this article.

The proof of the relevant stability criteria usually takes advantage of the comparison principle, which is judged by the property of a deterministic system.

Sampled-Data Control

Consider the following MJS:

$$\dot{x} = (A(r(t)) + \Delta A(r(t)))x(t) + B(r(t))u(t) \tag{20a}$$

$$y(t_k) = C(r(t_k))x(t_k), \quad x(t_0) = x_0, \quad r(t_0) = r_0, \tag{20b}$$

where $\Delta A(r(t))$ is the uncertain matrix with specified structure.

The sampled-data controller has the form

$$u(t) = F(r(t_k))y(t_k), \quad t \in [t_k, t_{k+1}), \quad k = 0, 1, \ldots. \tag{21}$$

Consequently, the closed-loop system has the form

$$\dot{x}(t) = (A_i + \Delta A_i)x(t) + B_i F_j C_j x(t_k), \tag{22}$$

where C_j is the specified $C(r(t_k))$ with $r(t_k) = j$. The closed-loop system described above is hybrid, in the sense that it consists of a continuous-time state $x(t)$, a discrete-time control action $F_j y(t_k)$, and a discrete-state Markov process.

Sampled-data control for MJSs has yielded fruitful results. To name a few, they include a dissipative-based adaptive reliable controller that was designed for systems subject to time delay, actuator failures, and time-varying bounded sampling intervals [102]; event-triggered reliable control for MJSs that are subject to nonuniform sampled data [103]; optimal sampled-data state feedback controller for continuous-time linear MJSs that was designed for H_2 and H_∞ performances [104]–[106]; sampled-data H_∞ filtering for singularly perturbed MJSs that were considered where time-varying delay and missing measurements were taken into account [107]; sampled-data control that was studied in the passivity-based robust framework for continuous-time MJSs [108]; and sampled-data control that was investigated in the passivity-based resilient control framework and adaptive fault-tolerant mechanism for MJSs subject to actuator faults in [109].

Conclusion

We provided a brief tutorial and survey on the stability analysis and control approaches for MJSs, which are of both theoretical and practical importance. This article's organization is unique, in that it contains both fundamental concepts for beginners and state-of-the-art research progress for experts.

MJSs should and will receive more attention in the future as advanced control techniques, wireless communications, and embedded computational units converge. These developments in multiple fields, and especially their convergence, naturally yield complex systems that contain both dynamic states and discrete events, thus leading to MJS models. In this sense, the study of MJSs will be of great help to further develop many intelligent systems such as intelligent transportation systems, the Internet of Things, and smart factory. More work is needed to address all of the new challenges for MJSs as we move toward the Industry 4.0 era.

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